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UDK: 336.711
DOI: 10.2478/jcbtp-2020-0040

Journal of Central Banking Theory and Practice, 2020, 3, pp. 123-134
Received: 11 March 2019; accepted: 27 June 2019

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Monetary Base Controllability after an Exit from Quantitative Easing[1]

Abstract: This study examines the problem that a central bank may face after exiting a monetary quantitative easing policy. It develops a simple dynamic optimization model of a central bank, which finds that if the bank needs to absorb a substantial amount of excess reserves when exiting, the monetary base may become uncontrollable. In this case, the bank has no option but to increase the monetary base by more than the target amount, which leads to an undesirable money supply expansion and, ultimately, to inflation pressures. The model shows the condition when a central bank faces such a challenging situation.

Keywords: central bank, monetary base, quantitative easing, exit strategy, solvency.

JEL Code: E52, E58

1. Introduction

The purpose of this paper is to examine the problem that a central bank may face after exiting a monetary quantitative easing policy. This paper develops a simple dynamic optimization model of a central bank and shows that a bank's unsound balance sheet after the exit may make it challenging for the bank to avoid an undesirable monetary base expansion.

[1] I would like to thank Takuma Kunieda, Kenjiro Hirayama, and the participants at the Financial System Workshop in Osaka for their helpful comments and suggestions. The responsibility for any remaining errors is mine alone.

After the "Lehman shock" in 2008, many central banks in industrialized countries introduced unconventional measures of monetary easing policy. One of the main measures was quantitative easing, where central banks expanded their balance sheets and excess reserves drastically. When an economy recovers, a central bank should exit quantitative easing using an exit strategy, such as substantially shrinking the balance sheet and paying high interest on piled up excess reserves.

There is a concern that exiting quantitative easing may deteriorate a central bank's balance sheet. Stella (2003) emphasizes the importance of a central bank's sound balance sheet, and Klüh & Stella (2008) and Adler, Castro & Tovar (2012) show that central banks with weak balance sheets do not perform well. Ize (2005) studies the financial problem of troubled central banks. Reis (2013, 2015), Hall & Reis (2015), Del Negro & Sims (2015), Berentsen, Kraenzlin & Müller (2016), and Fujiki & Tomura (2017) study financial stability of some central banks with various exit strategies for quantitative easing under several rules concerning dividend or fund transfers to and from governments.

These studies take into account the effect of a central bank's profit on the balance sheet, but they do not drive the bank's behaviour from optimization. This paper develops a continuous time version of the dynamic optimization model proposed by Tanaka (2014). Using the model, this paper shows the optimal behaviour of a central bank after exiting quantitative easing, and shows the possibility that a bank might be forced to expand the monetary base more than necessary.

This paper is organized as follows. Section 2 overviews the situation of the Bank of Japan (BOJ) as an example of quantitative easing by the central bank and explains the available exit strategies. Section 3 presents the model and Section 4 discusses certain policy implications of the model.

2. The Bank of Japan and Exit Strategies

2.1. Expanded Balance Sheet of the Bank of Japan

The BOJ first introduced an unconventional monetary policy in February 1999, which led the policy rate to zero. Since then, it has often implemented unconventional policies, including quantitative easing, and it is currently taking the most aggressive "quantitative and qualitative easing" that has started in April 2013.

Considering the unconventional monetary policy, the BOJ has expanded its balance sheet. Table 1 shows its balance sheet as at March 2018. The BOJ expanded its balance sheet by mainly purchasing Japanese Government Bonds (JGBs), while most of the supplied funds are piled up as excess reserves.

When the economy recovers and quantitative easing is no longer necessary, the BOJ should substantially reduce the excess reserves. Otherwise, private banks would start using the excess reserves for lending, which would lead to money supply expansion, and ultimately to inflation pressures.

Before the BOJ started using unconventional policies in February 1999, private banks held a negligible amount of excess reserves. In the decade from February 1989 to January 1999, excess reserve was only 0.432% of the total reserves on average. Thus, when the economy recovers, private banks would possibly use almost all of the excess reserves; thus, the BOJ should absorb them to prevent this from happening.

2.2. Exit Strategies

Several exit strategies of quantitative easing are available for the BOJ. One of them is shrinking its balance sheet by decreasing JGB holdings. JGBs can be redeemed, but it takes several years to wait for a substantial amount of their redemption. Thus, although the bank needs to sell JGBs, it imposes a large capital loss because the interest rate rises at the exit, which lowers the JGBs' price.

Bernanke (2009) proposes other exit strategies for such central banks so that excess reserves are not used for lending. One is to absorb the reserves by using fund-absorbing operation, such as reverse repo. The other is to pay interest on reserves, which is sufficiently high to prevent private banks from using the reserves for lending. However, these strategies also bring some losses to the central bank as the bank needs to pay high interest.

All of the above exit strategies deteriorate a central bank's balance sheet and reduce the bank's profit, possibly to negative. However, a sound balance sheet is important for central banks, and this has been studied by Stella (2003), Ize (2005), Klüh & Stella (2008), and Adler, Castro & Tovar (2012). Reis (2013, 2015), Hall & Reis (2015), Del Negro & Sims (2015), Berentsen, Kraenzlin & Müller (2016), and Fujiki & Tomura (2017) study financial stability of some central banks with various exit strategies for quantitative easing.

These studies take into account the effect of a central bank's profit on the balance sheet, but they do not drive the bank's behaviour for optimization. The rest of this paper constructs the dynamic optimization model of a central bank and studies its monetary base control after the exit.

3. Dynamic Optimization Model of a Central Bank

3.1. Balance Sheet and Profit

The model in this paper uses the simplified balance sheet of a central bank shown in Table 2. The central bank has assets A on the left side of the sheet, and has only the monetary base H and capital K on the right side. The bank needs to absorb RA out of H to exit quantitative easing.

The bank has two exit strategies. One is to sell RA amount of assets, which shrink A and H by RA. The other is to absorb RA by using fund-absorbing operation, such as reverse repo. Let such a liability be denoted by L, then RA is replaced by L and H is decreased by L. If L is the part of reserves with high interest rate instead of reverse repo, then it can be considered as the exit strategy to pay high interest on reserves.

The bank receives revenue from interest return. The net interest return for the bank is $r_A A - r_L L$, where r_A and r_L are the interest rates on A and L, respectively. For simplicity, $r_A = r_L = r$ is assumed; thus, by letting $a = A - L$, $ra = r_A A - r_L L$. The bank should pay its operating cost C, and thus its profit π is

$$\pi = ra - C, \tag{1}$$

where r and C are assumed to be constant over time in this paper.

In many countries, a central bank's profit is transferred to the government. For example, Japan has the basic rule that the BOJ adds 5% of its profit to the capital, while the rest is paid either as dividend to shareholders or as a transfer to the government. However, this rule assumes that the profit is positive and it is not clear whether the BOJ can receive any fiscal support in the case of negative profit. In addition, there is no clarity on how much fiscal support the Federal Reserve or the European Central Bank can receive, as pointed out by Reis (2015, p.3)[2]. The model here can easily include such fiscal transfers and support, but assumes that there are none. The full amount of π is added to K:

[2] Some central banks have a clear rule for fiscal support. For example, the National Bank of the Republic of Macedonia receives fiscal support not to have its capital reduced (Bezhoska, 2017).

$$K' = \pi = ra - C, \tag{2}$$

where $K' = dK/dt$ and t is time.

From the balance sheet and equation (2), the following is derived:

$$a' = H' + K' = h + ra - C, \tag{3}$$

where $a' = da/dt$ and $h = H' = dH/dt$. Equation (3) shows how the balance sheet is affected over time.

3.2. Dynamic Optimization

The model in this paper examines the dynamic optimizing behaviour of a central bank after the exit. The bank exits quantitative easing and shrinks A or expands L to a_0 at $t = 0$. After the exit, the bank controls the monetary base to keep the inflation at a desirable rate. Let h^* be the monetary base increase target that is consistent with the desirable inflation, such as 2%. Then, the bank minimizes the following quadratic loss function;

$$\min_h \int_0^\infty e^{-\delta t}\left\{\frac{1}{2}(h - h^*)^2\right\}dt \tag{4}$$

$$\textbf{s.t. } \boldsymbol{a}' = \boldsymbol{ra} + \boldsymbol{h} - \boldsymbol{C}, \tag{3}$$

$$a = a_0 \text{ at } t = 0. \tag{5}$$

Given δ, r, C, h^*, and a_0, the central bank attempts to set h for all $t \geq 0$ close to the target h^*.

Without the restrictions (3) and (5), the bank can fully control the monetary base, and thus it sets $h = h^*$, which leads to the desirable inflation. With the restrictions, the bank may face the case where it needs to set $h > h^*$, which accelerates the inflation., as discussed in the next section.

h is a control variable and a is a state variable. The current value Hamiltonian H is

$$\mathcal{H} = \frac{1}{2}(h - h^*)^2 + m(ra + h - C), \tag{6}$$

where m is the current value multiplier. The first order conditions are as follows:

$$\partial\mathcal{H}/\partial h = h - h^* + m = 0, \tag{7 – 1}$$

$$m' = \delta m - \partial\mathcal{H}/\partial a = (\delta - r)m. \tag{7 – 2}$$

Equations (7-1), (7-2), and (3) reduce to the next two differential equations.

$$a' = ra + h - C, \quad (8)$$

$$h' = (\delta - r)(h - h^*). \quad (9)$$

A steady state (h_S, a_S) is

$$h_S = h^*, a_S = \frac{C - h^*}{r}. \quad (10)$$

Graph 1 illustrates a phase diagram of the variables h and a. The $a' = 0$ locus is derived from equation (8) and its slope is $-1/r$. a is increasing in the area to the right of the $a' = 0$ and is decreasing in the area to its left. The $h' = 0$ locus is derived from equation (9) and it is vertical at h^*. h is decreasing if $h > h^*$ and is increasing if $h < h^*$. The steady state (h_S, a_S) is a saddle point. A central bank needs to satisfy the no-Ponzi game condition,

$$\lim_{t \to \infty} e^{-rt} a \geq 0, \quad (11)$$

in order to stay solvent, as discussed by Reis (2015).

The optimal paths of a are derived from equations (8) and (9) as follows:

$$a = \alpha_1 e^{rt} + \alpha_2 e^{(\delta - r)t} + a_S. \quad (12)$$

As $1 > r > \delta > 0$ is assumed, only the path with $\alpha_1 = 0$ can reach a_S.

With the restriction (5), the optimal paths of a and h to the steady state are derived as follows:

$$a = (a_0 - a_S)e^{(\delta - r)t} + a_S, \quad (13-1)$$

$$h = -(2r - \delta)(a_0 - a_S)e^{(\delta - r)t} + h^*. \quad (13-2)$$

These equations are illustrated as the convergence locus in Graph 1. Its slope is $-1/(2r - \delta)$, which is smaller than the slope of $a' = 0$ locus in absolute value.

4. Policy Implications

4.1. Case of $a_0 < a_S$

Using the model constructed in Section 3, this section examines the optimal path of the monetary base and checks whether or not a central bank can avoid undesirable monetary base expansion after exiting quantitative easing. At the exit, the bank shrinks a to a_0. This subsection examines the case where the bank needs to shrink a substantially to a point below a_S

In Graph 1, such a_0 is shown as a_0^1. As h is a control variable, the bank can choose any value on the horizontal dotted line at $a = a_0^1$. If the bank sets $h = h^*$, then a starts moving toward negative, and the condition (11) does not hold. In order to avoid a decrease in a, the bank must increase h to reach the $a' = 0$ locus, but it is not optimal either. The bank's optimal behaviour is to increase h further to the convergence locus. The no-Ponzi game condition is binding, and (11) with only an equal sign holds. It is now the transversality condition for the model, and the solution is shown through equations (13-1) and (13-2). The bank's optimal solution only gradually moves along the convergence locus toward the steady state point (h_S, a_S).

Therefore, a central bank cannot maintain a monetary base increase at the target h^* after the exit. It has no option but to continuously increase the monetary base faster for a certain period, which leads to money supply expansion and, ultimately, to inflation pressures.

The rate of return on net assets r is given in the model, but it may become lower after the exit if the bank chooses an exit strategy to absorb the reserves by using reverse repo or to pay high interest on reserves. This is because the assets obtained before the exit earn low interest, while new liabilities after the exit cost high interest. It is expressed as an exogenous fall in r. It raises a_S and makes the convergence locus flatter. Both of them force the bank to set h higher, and the situation deteriorates.

The model assumes no fiscal support from the government to the central bank. In the model, if the bank receives fiscal support, which amounts to $h - h^*$, the bank can maintain h at h^*. Thus, in the case of full fiscal support, the model shows its necessary amount[3].

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[3] Such fiscal support raises another problem; it weakens a central bank's independence as some literature such as Ivanović (2014) and Jasmine, Mona, & Heba Talla (2019) emphasize. With weak independence, the bank may receive pressure from the government to accelerate the monetary base increase and thus to cause higher inflation as many studies confirm (Fabris, 2018).

4.2. Case of $a_0 \geq a_S$

Suppose the central bank need not significantly shrink its balance sheet such that a_0 is above a_S. In Graph 1, such a_0 is shown as a_0^2. If the bank sets $h = h^*$ for all $t \geq 0$, the minimal value of loss function (4) is achieved. Equation (8) at $t = 0$ is

$$a' = ra_0^2 + h^* - C \geq ra_S + h^* - C = 0,$$

as equation (10) holds. a starts with a positive value at $t = 0$, and it never decreases throughout $t \geq 0$. a stays positive, and the condition (11) is not binding. Therefore, in contrast to the preceding subsection case, the bank can always set h at the target h^*.

The discussion in this subsection shows that central banks need not be concerned about their balance sheet's soundness in normal times. Generally, seigniorage results in a strong balance sheet, bringing more a with less H, and thus a is far above a_S in normal times. The monetary base is controllable as is assumed in many researches on monetary policy.

4.3. Case of the Bank of Japan

The model constructed above is now applied to the case of the BOJ described in Section 2. Table 1 shows the amount of the net assets is $a = A - L = 529 - 86 = 443$ trillion yen. For the fiscal year ending March 2018, the profit except the operating cost ra is 1.42 trillion yen and the operating cost C is 0.195 trillion yen. The rate of return on the net assets is $r = 1.42/443 = 0.321\%$, and even if $h^* = 0$, $a_S = (C - h^*)/r = 60.7$ trillion yen. On exit, if the BOJ absorbs all the 320 trillion-yen excess reserves, then $a_0 = 443 - 320 = 123$ trillion yen. Thus, as long as r does not fall, a_0 is well above a_S, and the BOJ has no difficulty in controlling the monetary base after the exit.

However, it may be too optimistic to assume that r does not fall. It may fall as the BOJ has often been purchasing JGBs with negative return since 2016 and since the bank may need to pay a higher interest rate on its liabilities to absorb the excess reserves. If the rate of return on the net assets falls lower than 0.158%, then a_S increases above a_0, and thus the BOJ starts facing challenges in keeping the monetary base unchanged.

5. Conclusion

This paper has examined monetary base controllability after an exit from quantitative easing. A simple dynamic optimization model of a central bank has been developed, and it has been used to derive a bank's optimizing behaviour. The analysis in this paper provides the followings findings.

First, if a central bank needs to absorb a substantial amount of excess reserves at the exit, then there is a case that it has no option but to increase the monetary base by more than its target amount; the monetary base is not controllable. This is because issuing monetary base brings seigniorage to the bank, which helps the bank to build up its assets to a sufficient level. The model shows the condition when a bank faces this case. The model does not assume any fiscal support from the government, but if the bank receives such a fiscal support that equals the amount of the optimal monetary base increase above the target, then it can keep the monetary base increase at the target.

Second, if the amount of excess reserves that the bank needs to absorb is not substantial, it can always keep the monetary base increase at the target. This shows that the monetary base is controllable in normal times as is assumed in many researches on monetary policy.

The model of this paper is a simple one and can be easily expanded to include various elements, such as different interest rates on assets and liabilities, and fiscal transfer. However, these remain for future studies.

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Table 1: The BOJ's Balance Sheet

Assets 529 (JGBs 427)	Bank Notes	104
	Required Reserves	10
	Excess Reserves	320
	Other Liabilities	86
	Capital	8

Table 2: Simplified Central Bank Balance Sheet

Assets (*A*)	Monetary Base (*H*)	
		Reserves to be absorbed (*RA*)
	Capital (*K*)	

Notes: Trillion yen.
Reserve figures are the averages of the reserve period, March 2018.
The others are at the end of March 2018.
Source: The Bank of Japan Homepage.

Graph 1: Phase Diagram of *h* and *a*