CENTRAL BANK OF MONTENEGRO

Working paper no. 31

ASSESSMENT OF POTENTIAL GROWTH OF MONTENEGRO`S ECONOMY USING THE PRODUCTION FUNCTION AND THE KALMAN FILTER METHODS

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Podgorica, 2022

PUBLISHED BY: *Central Bank of Montenegro*
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1. Introduction

Assessing potential economic growth (output) is an important input for macroeconomic analysis. Potential output is the level of gross domestic product (GDP) that an economy could generate if it operated at its full capacity. Given that potential growth cannot be directly measured and that it is an abstract concept, it should be estimated based on available data and using various statistical and econometric methods. Potential output estimation tools include various time series filters, as well as theory-based methods, e.g. the production function method. In the context of countries in transition, these methodological approaches have certain limitations and, therefore, they should be used in a complementary manner.

This working paper presents the results of the assessment of potential output and output gap for Montenegro in the period from 2006 to 2023. The results were obtained using two methodological approaches: the production function method and the Kalman filter method. The estimate of the potential output using the production function are to be obtained by calculating the trend of individual production factors. This approach allows us to identify the contribution of individual productions factors and provides a good basis for identifying supply-side bottlenecks that hamper economic growth and progress. The production approach is based on a theoretical framework although its aplication is somewhat more difficult due to the absence of certain time series. On the other hand, the Kalman filter is an algorithmic method that is based on an algorithm. The obtained results show a strong negative impact of the crisis caused by the coronavirus pandemic on potential output, thus indicating that the negative gap will continue in the next two years, but the real GDP will be very close to the potential GDP. The results are as expected becase usually after major crises, developing economies face challenges to stimulate potential growth. Empirical findings in the literature indicate that deviations of GDP from its potential remain present for years following a crisis and/or a certain shock that caused the crisis. The results of the assessment in this paper suggest that real GDP could catch up with potential GDP in 2023.

This paper provides a simple framework for thinking about potential economic growth by looking at different channels through which potential economic performance can be influenced. Adequate assessment of potential growth and the information on the size of the GDP gap are particularly important for economic policy makers in the context of the crisis caused by the coronavirus pandemic. However, it should be noted that the estimates are subject to some uncertainty given that the pandemic is ongoing and certainly affects potential output.

The paper starts with the introduction after which the second part explains the concept of potential output in macroeconomic analysis and the methods for its calculation. The third part explains the production function method used to provide the calculation of potential output and output gap. The fourth part of the paper presetnts the methodology for calculating the potential output using the Kalman filter and the results of this approach. The paper ends with the conclusions.

2. The concept of potential output and measurement techniques

Potential output is the level of GDP that an economy could generate if it operated at its full capacity. In other words, potential output is defined as the level of production that an economy could sustain when the production factors at the current level of technology are fully deployed. Full utilisation of production factors, capital and labour is considered the maximum production that does not cause inflationary pressures (Okun, 1962), i.e. it has a sustainable employment rate. The production gap is the difference between the actual level of production and the level of production with full use of the production factor. Therefore, a positive gap in production can be considered a measure of the excess aggregate demand over aggregate supply. As a result, a positive production gap can also serve as an indicator of inflationary pressures in the economy. In contrast, negative production gap values signal excess production capacities and declining inflationary pressures.

The concept of potential output is of great importance for short-term and long-term macroeconomic analysis. In the short run, the output gap indicator provides a mean value of short-term transient impacts (Giorno et al., 1995), while in the long run, potential output indicates a sustainable position of economic growth that creates non-inflationary pressures (Rõõm, 2001).

By estimating potential output and the corresponding output gap, we can identify any increase in underlying imbalances or structural positions in macroeconomics (Giorno et al., 1995). Moreover, the assessment of actual performance in relation to the potential provides guidelines for calibration of both fiscal and monetary policies. If a country`s trend of potential growth lags behind the neighbouring countries (comparable economies), that would require structural reforms to improve the business environment in that country. Namely, it is assumed that structural reforms are the driving force of economic growth (Berg et al. 1999, Kolodko 2000, Fischer et al. 2000).

Given that potential output and the output gap are not directly visible and measurable, and having in mind the difficulties related to data in transition countries, this paper analyses the most appropriate techniques for estimating potential output recommended by the empirical literature. What we need to keep in mind is that in the context of transition economies, the output gap can be subject to large and constant structural adjustments and changes, often reflected in high structural unemployment rates, imported inflation and various market failures that continuously affect and usually limit potential production (Kastrati et al., 2017). Therefore, the output gap in transition countries such as Montenegro may reflect not only changes in actual output but also in the potential output.

Different methods are used to estimate a potential output and the corresponding output gap. Potential output estimation tools include univariate statistical filtering methods (Hodrick and Prescott, 1997; Baxter and King, 1999; and Christiano Fitzgerald, 2003); followed by the state space models that include the Kalman filter technique, and theory-based methods such as multivariate models (Blagrave, 2015) and the production function (Cobb - Douglas) approach.

All methods used to estimate potential output rely on detrending techniques, i.e. decomposing or filtering the observed macroeconomic variable (actual output) into unnoticed variables (components): trend, cycle and corresponding inconspicuous parameters, which should be subsequently assessed.

$$y_t = \tau_t + c_t$$

τ_t – long term trend component

c_t – cyclic component (deviations from the long-term trend)

The filtering procedure requires the extraction of a number of unnoticed parameters that are usually included in the noise (e.g. trend and cycle variance). When using univariate techniques, the task is to extract many unnoticed variables and parameters from the noise using only one visible variable (real GDP), which can be rather an ambitious endeavour.

For instance, the Hodrick-Prescott (HP) filter is a two-sided symmetric moving average filter, which assumes that trend and cycle components are not correlated and that the trend is a "smooth" process without any sudden changes over time.

Univariate methods have the advantage of being simple and easy to interpret, which makes them widely used. However, some univariate models (such as the HP filter) suffer from a number of shortcomings, which usually makes them less accurate compared to multivariate methods. For example, they suffer from endpoint issues, which implies excessive sensitivity to the latest data. In essence, the HP filter assumes that the initial and final variations are mostly trend changes and not cyclical variations. Endpoint limitation leads to frequent revisions of historical estimates of the output gap, with the greatest degree of uncertainty reserved for the last and most needed data point required to make appropriate economic policies.

Also, as often mentioned, many transition economies tend to be subject to structural changes that cannot be identified by univariate filtering techniques such as the HP filter (Kastrati et al., 2017). Matching the mean trend to real data, with the cycle below the real effect as much as above, may not be appropriate in the context of transition. Namely, other methods are needed that would enable the matching of a different mean value. Multivariate methods have a characteristic advantage in the fact that they use theoretical bases to determine unnoticed indicators, i.e. potential output and output gap. Using the production function, we can observe changes in the supply-side performance based on observed developments in the labour volume, capital, and total factor productivity. For instance, if an increase in the rate of capital growth is accompanied by an increase in the trend of total factor productivity, we can indicate a certain improvement in performance on the supply side. On the other hand, if we notice an increase in the capital growth rate while the trend of total factor productivity remains stagnant, we will conclude that the supply side is functioning inefficiently. Hence, it is often concluded that the production function is a useful tool for macroeconomic analysis and evaluation of government structural policies.

To summarize, each of the presented methods has its benefits and drawbacks, and the choice of the appropriate method for countries in transition is usually conditioned by data. When it comes to small and open economies such as Montenegro, this is not an easy task because the available time series are short and not all data necessary for a complete analysis are available.

The production function method makes it possible to identify various factors that contribute to the growth of potential output. However, this method can be used with data from a rather limited period of time, with quite sensitive assumptions, especially in terms of technological progress and capital valuation.

3. Production function

The production function in Montenegro is estimated based on data from the period 2006 - 2020. Total production is expressed in gross domestic product. The production function approach provides the output that is closely correlated with technological progress, employment and the level of capital in the country. So the gross domestic product is obtained from the interdependence of the three variables: (1) labour input (Lt); (2) capital used (Kt); and (3) efficiency used by these factors, that is, total factor productivity (At).

The relations between these three variables can be shown as follows:

$$Y_t = A_t L_t^{\alpha} K_t^{1-\alpha} \quad (1)$$

This equation is called the Cobb-Douglas production function. Taking the logarithm on both sides of the equation, we get the following expression:

$$y_t = a_t + \alpha l_t + (1 - \alpha) k_t \quad (2)$$

Assuming the Cobb-Douglas production function, it is known that the elasticity of α labour input to GDP is equal to the share of labour in equilibrium. Therefore, using equation (2), it is easy to calculate the change in total factor productivity (at), or the growth rate At, from the observed change in yt, lt, kt, which is the growth rate Yt, Lt, Kt. The same formulation of the production function can be applied to the equation for potential GDP as follows:

$$Y_t^* = A_t^* L_t^{*\alpha} K_t^{*(1-\alpha)} \quad (3)$$

Similarly, as in equation (2), the logarithm of both sides of the equation gives the following:

$$y_t^* = a_t^* + \alpha l_t^* + (1 - \alpha) k_t^* \quad (4)$$

If we subtract equation (4) from (2), because the output gap is defined as the difference between real GDP (Yt) and the potential GDP (Yt^*), we get:

$$y_t - y_t^* = (a_t - a_t^*) + \alpha(l_t - l_t^*) + (1 - \alpha)(k_t - k_t^*) \quad (5)$$

$$gap_t = \alpha(l_t - l_t^*) + (1 - \alpha)(k_t - k_t^*) + \varepsilon_t \quad (6)$$

where $\varepsilon_t = (a_t - a_t^*)$ is defined as TFP noise or the so-called Sol residue, i.e. output reduced by the weighted sum of labour and capital inputs.

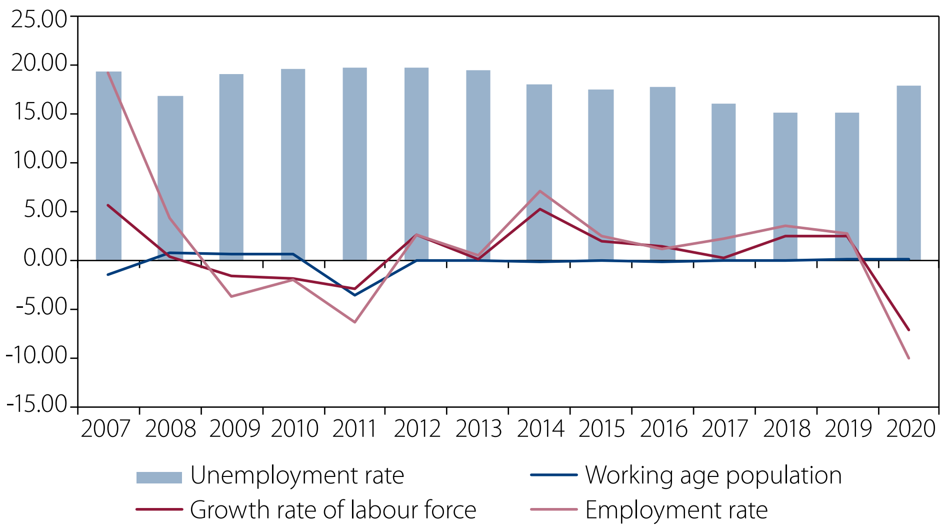
Using this method, we will first calculate the total factor productivity (At) as a residual of the calculation formula for growth for the period from 2006 to 2020, while we will make assumptions about the growth rates of capital and labour for the projected period from 2021 to 2023. The basic assumption is that the weights of labour participation in production and capital are 65% and 35%, respectively, while the annual rate of depreciation is 5%. Assumptions were obtained based on the findings of research by D'Auria et al. (2010) and Bosworth and Collins (2003) who indicate that the labour force share (α) is set at the EU average of 65 percent. Since data on the value of capital in Montenegro is not available, an assessment was made on the basis of previously conducted empirical research. The capital assessment was done in line with the methodology of accumulation of gross fixed capital formation in the base year minus the the capital value decrease over time. This methodology, known as the fixed inventory method, requires the following data: the value of share capital in the initial year, a continuous time series for the value of investments (i.e. gross fixed capital formation), and the extent to which capital decreases in value over time. The process of capital accumulation is shown in the following formula:

$$Kt = Kt{-}1(1 - \delta) + It\,, \quad (7)$$

with It indicating the level of real investment in each period and δ depreciation of Tier1 capital in the previous period. We use a rate of 5 percent for capital depreciation (delta).

We use employment to approximate the labour invested in the production function. In the chart below we can see that we had positive employment rates during the years of economic growth, as a result of increased labour demand. During the boom period, traditional labour-intensive activities such as construction, mining, and retail trade were on the rise. However, while employment reacted positively to economic growth, the unemployment rate was still very high, even in periods of rapid economic growth. A high unemployment rate indicates that there is a high structural component that the business cycle is not able to neutralize. Although the increase in economic growth, especially during the investment boom recorded from 2017 to 2019, led to a decrease in the unemployment rate, this rate was still high compared to the neighbouring countries and the EU Member States.

Graph 1 – Growth rates of employment, working age population, labour force participation, and unemployment



Source: MONSTAT

It is the high unemployment, even during a period of rapid growth, which indicates that the economy is functioning below its potential. The limiting factor in reducing unemployment in Montenegro is the so-called structural unemployment which is conditioned by the mismatch of supply and demand of professionals for certain job profiles. Investment cycle in the period 2016 - 2019 did condition the demand for certain professionals in the field of construction, tourism, agriculture, and energy but restrictions on the supply side were still present.

Relatively low labour force participation rates as well as high structural unemployment on the supply side indicate that a high percentage of the population does not benefit from an active labour market. If we look at the year before the COVID-19 pandemic, 2019, less than half of women of active age were in the labour force, while the youth unemployment rate was about 25%, which was significantly higher than in the European countries. In the pandemic year, the indicators further deterirated.

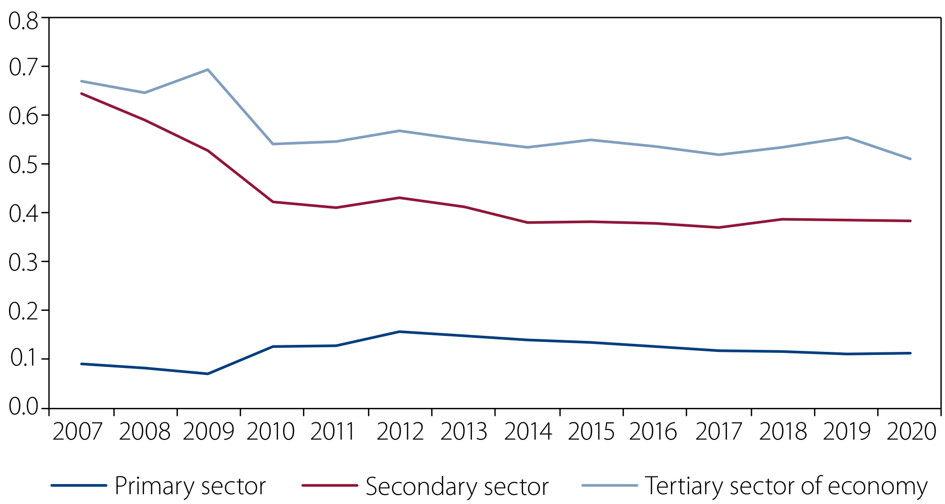
Unfavourable demographic situation additionally contributes to the rigidity of the labour market in Montenegro. Low birth rates and increased life expectancy lead to rapid population aging. Montenegro has a birth rate below simple reproduction of 1.7 and an increase in life expectancy. As shown in the graph 1, the share of the population aged 15+ has not changed since 2012.

Due to the lack of data, no correction was made in the labour input to change the actual working hours spent. Also, for the same reason, the labour component cannot be differentiated in terms of quality resulting from a certain level of education. However, using the method presented in the work of Rõõm (2001), we will point out the differences in productivity in different sectors of the economy (primary, secondary, and tertiary sector). Namely, we calculate an index that describes the total change in productivity, generated by the redistribution of labour. The index has higher values when people move to more productive sectors, assuming that real earnings can be used as an approximation of productivity. Also, it is assumed that relative productivity in the sectors is constant over time. The index is calculated using the following formula:

$$h_t = \sum_j \frac{\frac{1}{T}\sum_{t=1}^{T} w_{t,j}}{\overline{w}} * \frac{L_{t,j}}{L_t} \tag{8}$$

where h is an index that describes the redistribution of labour into more productive sectors, w – real earnings, $\overline{w}$ – average real earnings, L – number of employees, T – number of periods, indices t and j describe the period, and i describes the sector.

Graph 2 – Labour participation in primary, secondary and tertiary economies

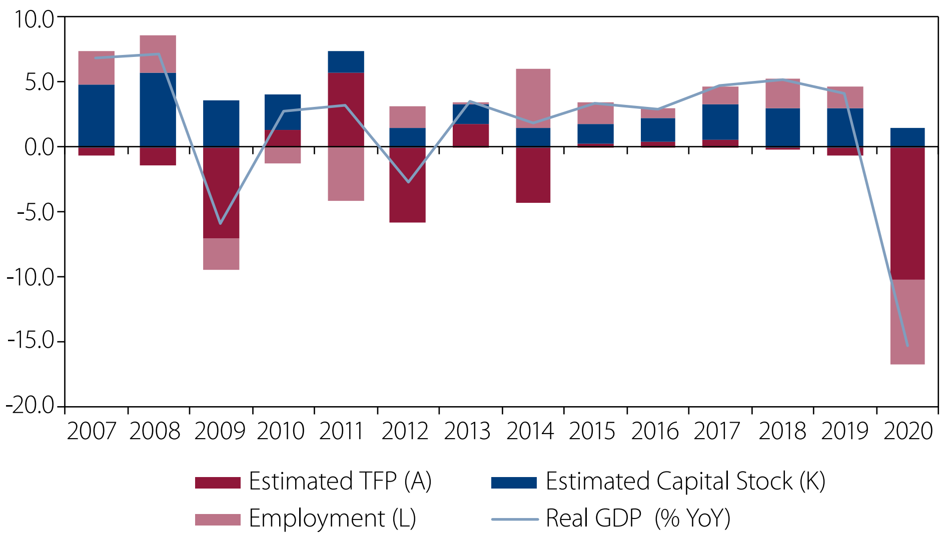


Source: Authors` calculations

Looking at the graph above, we can notice a stable prevailing share of the labour force in the tertiary sector of the economy. The share of labour in the secondary economy has a downward trend, while we also notice low employment in the primary sector, which was on a mild donwtrend in recent years.

By applying the mentioned assumptions, we calculate the total factor productivity for the period 2006 – 2020, i.e. the third component of the production function and the results are presented in the graph below.

Graph 3 – Contribution to the growth of the economy based on the production function, 2006-2020



Source: Authors` calculations

Growth averaged at 2.8% in the period 2006 - 2019 (or 1.6% with 2020 included). In the observed period, capital participated with 2.66 pp annually, labour with 0.92 pp, and total factor productivity (hereinafter: TFP) accounted for 0.75 pp per year. If we were to include the year 2020 in the observed period, capital would contribute 2.58 pp annually, labour 0.39 pp, while TFP would account for -1.42 pp per year.

A negative TFP contribution is unusual. What is particularly surprising is that the TFP is not only negative in crisis times, such as the global financial crisis and its reflection on the crisis processes in Europe in 2012. The low historical growth of TFP can be explained by a long process of economic transformation and slow structural reforms, which caused the technological obsolescence of a part of the capital and the slow development of organisational (managerial and functional) capabilities of economic entities. In her working paper, Vujanović (2020) uses micro data (at the company level) to measure the level of total factor productivity (technological efficiency) in the services sector and the manufacturing sector of Montenegro in the period 2010-2019. The results indicate that the manufacturing sector, which is considered the sector with the greatest potential for technological growth, did not record an increase in total factor productivity, and that the TFP trend was relatively stable from 2013 to 2019. Only industries that did not require high technologies experienced growth of technological efficiency. Also, during the ten-year period, total factor productivity of the services sector, which contributes a significant 40% of GDP (MONSTAT, 2019), remained stagnant. The referenced paper points out that there is a lot of room for improving the total factor productivity in Montenegro and that digitalisation, which is important for the improvement of technologies – is not widespread in the country. There is room for improving total factor productivity, especially in the field of high-tech manufacturing and service industries that require a high level of expertise such as IT, telecommunications, scientific research and development, and the like.

The next step is to estimate the potential GDP. Specifically, the average inputs of labour (lt*), capital (kt*) and TFP trend (at*) are estimated from the observed data, thus the potential GDP (yt*) is given in equation (4).

The key input for calculating potential output and output gap is potential employment. According to the production method, the idea is to find a level of employment that does not accelerate inflation. One approach is to approximate it using the natural Non-Accelerating Inflation Rate of Unemoployment (NAIRU). In the production analysis, the labour market is concentrated on the unemployment rate, i.e. the way in which the unemployment rate is developing depending on the phase where we are in the business cycle. When the economy is at its "normal" or long-term level of production (i.e. the potential GDP), there is an unemployment rate known as the "natural" unemployment rate. This unemployment consists of frictional and structural unemployment, but there is no cyclical unemployment associated with business cycles. The Okun`s law implies production growth but also the structural unemployment rate due to slow adjustment of the labour market. Namely, it is known that the labour market is always behind the overall business cycle, i.e. that there is this persistence in explaining the labour market reaction.

Empirical works approximate potential employment using the unemployment level trend or long-term unemployment rate. The basic assumption in the empirical literature is that technological change causes a decrease in the skills of people who are not currently working, i.e. the longer a person remains unemployed, the more their qualification decreases (Rõõm, 2001; Kawamoto et al., 2017). Therefore, potential employment is calculated using the long-term unemployment rate.

Labour market projections are focused on the Okun-type unemployment rate, total labour resources determined by a given working age population (population aged 15 and over) and the participation rate of the working age population. The last two variables are given exogenously. The data from the MONSTAT Labour Force Survey were used for the period 2006 - 2020. For the projection period 2021 – 2023, projections of the UN population[1] were used, as based on probabil-

[1] *UN 2019 Revision of World Population Prospects*

istic projections of medium fertility and life expectancy at birth. These probabilistic projections of mean fertility and life expectancy at birth were conducted using the Bayesian hierarchical model (UN, 2019).

As for the unemployment rate, it depends on the NAIRU that we estimated by applying the HP filter to the long-term unemployment rate and the unemployment rate projection for the period 2021–2023, in line with economic growth projections for the observed period. Given the time period of observation and due to the lack of sufficiently long time series, in order to verify the estimated NAIRU, we took a long-term average unemployment rate for Montenegro that approximates the unemployment rate in the steady state.

Finally, when we have the estimated unemployment rate, an estimate of the participation of the working age population and an assumption of the expected available labour force, we can calculate the potential employment.

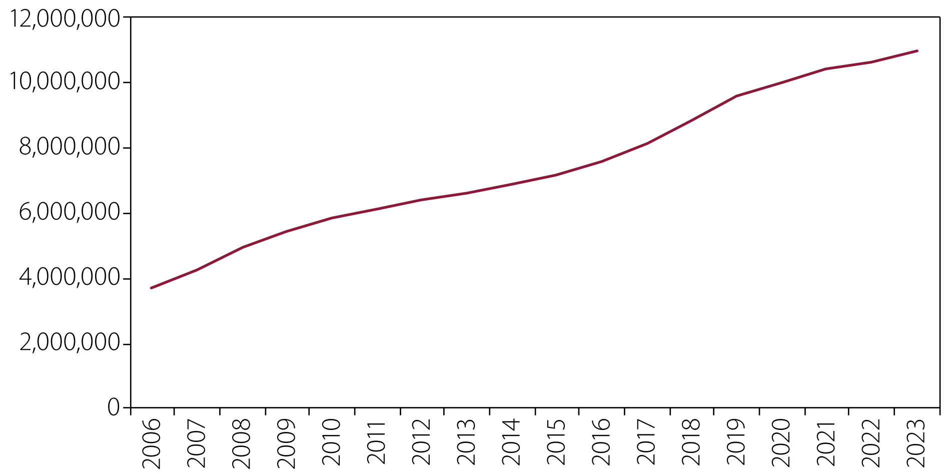
The working age population is multiplied by the participation rate (as all those who are part of the working age population are not part of the labour force), which is further multiplied by one minus NAIRU (as not all those in the labour force are employed).

Potential employment = potential labour force*(1-(NAIRU/100))

Potential labour force = working age population * HP filter for participation of working age population

It is assumed that capital is at its potential level.

Graph 4 – Capital trends from 2006 to 2023



Source: Authors` calculations

A strong investment activity in the period from 2016 to 2019 in transport, tourism, energy and agriculture further raised the value of capital, which will affect the growth of potential rates of economic activity. The investment in the first section of the highway gave a special impetus to capital growth.

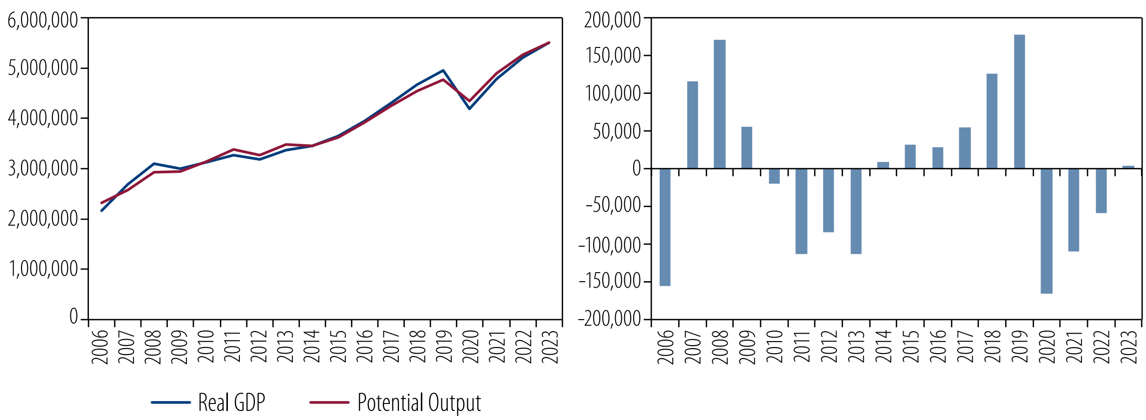
We will calculate the potential level of total factor productivity using the HP filter to find its trend level. The methodology for estimating the growth of the TFP trend (changes compared to the previous year $at*$) includes the following: first, the calculation of the year-on-year growth rate of the real TFP (changes at compared to the previous year), and then applying the HP filter (λ = 1600) on the growth rate of the real TFP.

An alternative approach is to assess technological progress using a simple trend or to approximate it through measures such as education, research and development costs. However, the lack of available data limits us from using this approach.

After estimating potential GDP, we will calculate the output gap that represents the difference between the potential GDP (yt*) and the real GDP (yt).

Potential gross domestic product, estimated by production function, and realised gross domestic product, are shown in Graph 5. The GDP series for the period 2021–2023 correspond to the forecasts of the Central Bank of Montenegro and the European Commission.

Graph 5 – Potential and real GDP (left) and output gap (right)



Source: Authors` calculations

The results of the production function indicate that Montenegro's GDP in the year of gaining independence was below its potential. The rapid economic growth from 2006 to 2008, supported by a strong inflow of foreign direct investments, caused a positive output gap, i.e. a positive difference between the real and potential GDP. With the onset of the global economic crisis and the negative GDP rate in 2009 (-5.8%), the production gap was declining, and in the period from 2010 to 2014 it entered the negative zone. From 2014 to 2019, an exponential growth of the production gap was recorded. At that time, the positive GDP gap (i.e. the positive difference between real and potential GDP) was at the highest level. However, with the onset of the COVID-19 epidemic, the production gap abruptly narrowed.

Due to the simultaneous significant decline in real GDP in 2020 on one hand, and the growth of potential GDP on the other hand, the highest level of negative output gap was recorded in 2020. In addition to the positive projected GDP rates for the period 2021 - 2023, the negative GDP gap will remain both in 2021 and 2022. The estimate indicates that reaching the level of potential production or potential level of GDP can be expected in 2023.

We can conclude that the practical application of the production function method requires the creation of certain assumptions such as constant returns on the labour to capital ratio, and they are often stated as a significant limitation of this approach. These assumptions limit the elasticity of production in relation to labour and capital to values between zero and one, that is, their sum is equal to one. Also, it is assumed that technological progress changes unhindered over time, which may not be applicable in transition countries that are going through numerous structural changes (Sramkova et al., 2010). Finally, the production function approach continues to use statistical filters, such as the HP filter, to assess the trend in total factor productivity, which in turn carries with it the shortcomings of the measures we previously highlighted.

4. Kalman filter

In this section, we measure potential output by an alternative method - the Kalman filter method, which belongs to the state space model. Unlike the previous model, which is based exclusively on economic theory, the Kalman filter is more an empirical-statistical method, but with a clear theoretical approach in its setting (Oksanen, 2018). In a way, this method is more accessible for estimating potential output because there is no need to approximate the missing series, such as the capital level. However, unlike the production method, estimation of total factor productivity, an important component for understanding technological advancement, is not possible by using the Kalman filter method.

The Kalman filter generates a series of potential outputs by eliminating the noise in the series it filters in a way that minimizes the mean square errors of the estimated parameters (Kleeman, 1996). For the purpose of filtering the data, an algorithm is created that can consist of two sets of equations that define the algorithm. The first set of equations (the so-called measurement equation) expresses the GDP series as a function of potential GDP and the GDP gap. The second set of equations (the so-called state equations) define the process of data generation that ultimately forms the potential GDP and the GDP gap. Given that the potential output and output gap is measured on the basis of one known series (GDP) instead of multiple known series, this is an univariate model.

There are several ways to formulate the Kalman filter algorithm that generates two unknown elements. However, before formulating the algorithm itself, it is useful to analyse the nature of the time series (of the GDP) to be broken down into potential output and the output gap. The nature of the time series is assessed through the application of stationarity tests. If the tests indicate that the GDP series is stationary, the values of its distribution (expected value and variance) do not change, i.e. they are constant during the analysed period. The data series arbitrates around a certain value and has no defined trend. Otherwise, the series is non-stationary and has a certain defined trend.

The Augmented Dickey-Fuller (ADF) stationarity test indicates that the GDP series is a non-stationary series with a unit root[2] (Table 1). The null hypothesis of the ADF test is that the series is non-stationary, i.e. that it has a unit root. The impossibility of rejecting this hypothesis (p value> 0.05) confirms that the series is non-stationary. If the ADF test result rejects the null hypothesis (p-values <0.05), the GDP series is stationed. The test indicates that the GDP series is non-stationary (p value = 0.173). On the other hand, by differentiating the series, a stable stationary series is obtained (p-value = 0.00). Logically, the GDP series has a certain trend direction.

Table 1 – Stationary tests

ADF test applied on:	P probability value	Conclusion
Logarithm (GDP)	0.173	Series of GDP is non-stationary
Diferentiated logarithm (GDP)	0.00	Series of diferentiated GDP is stationary

This information is useful because the potential GDP series has the same features (stationarity status) as the real GDP series. This means that the potential GDP series is also a non-stationary series with a unit root.

After checking the stationarity of the series, the Kalman filter is generated by a series based on the model setting (algorithm), and the initial parametres of the model based on which the filtering of GDP data starts.

Model setting (algorithm)

When creating an algorithm, the equation of state should be formulated to define the potential output as a series that is non-stationary and with a certain trend. Therefore, we define a given equation of state as a autoregressive process of the first order with a constant.

Based on the above analysis, but also theory and practice, we opted for the following algorithm:

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[2] *The series has a unit root if the series needs to be differentiated in order for it to be stationed. In other words, if the GDP_t series (t-year) is non-stationary and has a unit root, $\Delta\ GDP_t$ (GDP_t - GDP_{t-1}) is the stationary series.*

$y_t = g_t + c_t$ MEASUREMENT EQUATION (9)

$g_t = \mu_g + g_{t-1} + \varepsilon_t \; \varepsilon_t \sim \text{iid}\,(0, \sigma_\varepsilon^2)$ EQUATION OF STATE 1 (10)

$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + v_t \; v_t \sim \text{iid}\,(0, \sigma_v^2)$ EQUATION OF STATE 2 (11)

The algorithm expressed by equations 9-11 is recursive and can be expressed in several ways (see Rummel, 2015). The equation 9 (measurements) expresses GDP (y_t) as a function of potential GDP (g_t) and the GDP gap (c_t). Hence, this equation breaks down the known GDP series into two unknown elements. The GDP series (y_t) is logarithmic. Equation (10) is the first equation of state that defines potential GDP as a autoregressive first-order process (random walk process with constant μ_g). ε_t and v_t are random equation of state errors (10) and (11), respectively, which are not mutually correlated. Also, these standard errors have a normal distribution with variance σ_ε^2 and σ_v^2, respectively. The parameters μ_g, ϕ_1, ϕ_2 are unknown parameters (regression coefficients).

Equation (11) expresses the GDP gap as a second-order autoregressive process. The recursiveness of this process is the result of both the potential GDP (g_t) and the GDP gap (c_t) expressed as a function of past values (g_{t-1} i c_{t-1} / c_{t-2} /, respectively). This means that the Kalman filter filters new values based on past values at time t, so that the GDP gap and potential GDP series are estimated in a number of consecutive steps, and after a large number of iterations, a potential GDP series is created. Thus, this series creation process is updated each time a new observation is entered into the algorithm.

Equations (9) to (11), along with their parametres, define the Kalman filter that filters the values of new series (potential output) based on unknown variables. In order for the algorithm of equations to be able to generate two series, assumptions about the initial values of the parametres μ_g, ϕ_1, ϕ_2, σ_ε^2 and σ_v^2 are required. Assumptions are given based on the theory (Kuttner, 1994), empirical analysis (Rummel, 2015), but also as an arbitrary process (the trial and error method). Like Kastrati et al. (2018), the parametre μ_g is a parametre that we equate with the average value of GDP, while the values of parametres ϕ_1 and ϕ_2 are taken from the theoretical framework (Kuttner, 1994) according to which the parametre of the first autoregressive (ϕ_1) is 1.4 and the second (ϕ_2) is -0.5, thus we meet the stationarity criterion according to which the sum of these two parameters is below 1. The other parametres were chosen by the trial and error method, but in a way that maximizes the maximum probability function. The initial parametres are shown in Table 2.

Table 2 – Initial algorithm parametres

Parametres	μ_g	ϕ_1	ϕ_2	σ_ε^2	σ_v^2
Values	7.9	1.4	-0.5	-10.3	-10

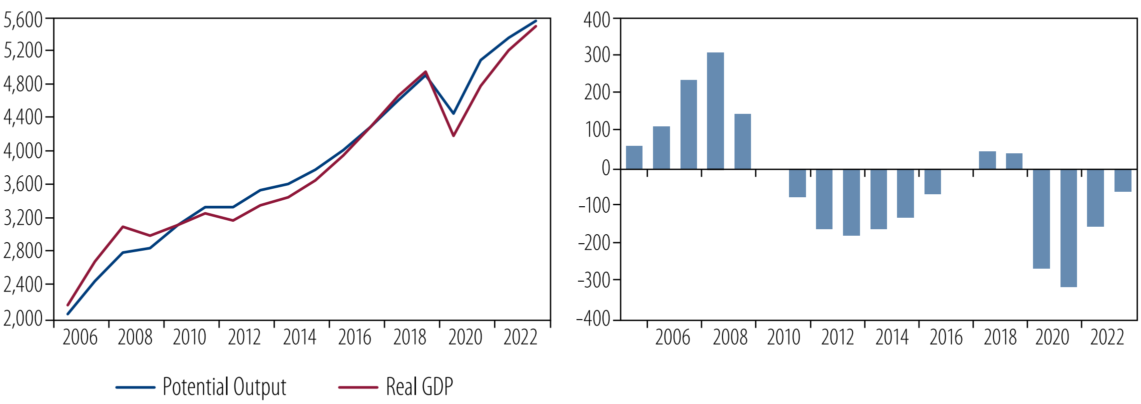
Source: Author's calculations

The results of the model (filtering) may differ, depending on the initial parameters but also on the algorithm settings, and this is the main drawback of the Kalman filter. Also, the output gap obtained by this method is not necessarily correlated with the level of inflation (Giorno et al., 1995). However, the potential benefits of this method are significant. The Kalman filter is suitable for filtering the series that have missing values through adjustements of the algorithm (Koopman and Ooms, 2002). In this sense, the Kalman filter can be used to predict future values and not just the missing ones. Also, some additional non-measurable variables, such as the natural level of unemployment, do not need to be used to estimate the potential GDP. In this particular case, there was no need to approximate the level of capital, which was missing from the production function model.

The model results

For the purpose of estimating the series of the potential GDP, the series of GDP at current prices[3] was used. Potential gross domestic product, estimated using algorithm 9-11, is presented in Graph 6 on the left-hand side, while estimation results are presented in the right-hand side of the Graph 6. The graph also shows the realised gross domestic product (at constant prices) during the analysed period. For the purposes of estimating the potential output, the available annual series of GDP expressed at constant prices in the period 2000–2020 was used, as well as the GDP forecasts of the Central Bank and the European Commission for the period 2021-2023. The Kalman filter results from 2006 onwards are presented for the purpose of comparison with the previous method.

Graph 6 – Potential output and real GDP (left-hand) and output gap (right-hand), the Kalman filter method



Source: Authors' calculations

The Kalman filter results give the expected result. The level of GDP is above the potential in the period from Montenegrin independence until 2010 when they are practically equal, which is the result of a large decline in GDP in 2009 (-5.8%). Namely, the positive GDP gap grows until the out-

[3] *In the earlier Central Bank of Montenegro working paper (Ivanović et al., 2021), we used a series of GDP at constant prices.*

break of the global financial crisis in 2008, and then declines. The high level of positive output gap can be explained by the high investment cycle that preceded the financial crisis.

In the period from 2010 to 2016, the Montenegrin economy produced below its potential, which indicates that the effects of the global economic crisis were long-term indeed. Due to the high growth rate in the period 2017-2019, which was above 4%, Montenegro again entered the zone of positive GDP gap. The explanation for the short-term return of the positive GDP gap can be found in employment growth, growth in consumption supported by a good tourist season, as well as more favourable credit conditions that encouraged investments. The construction and trade sectors also recorded significant growth.

However, the pandemic brought about a quick shift from a slightly positive to a very negative GDP gap. In 2020, both potential and real GDP suffered a sharp decline, with the level of real GDP falling significantly (-15.3%) due to a decline in the services sector, primarily tourism, which is an experience shared by many countries which base their production on services. Montenegro recorded the most negative GDP gap in 2020 since its independence.

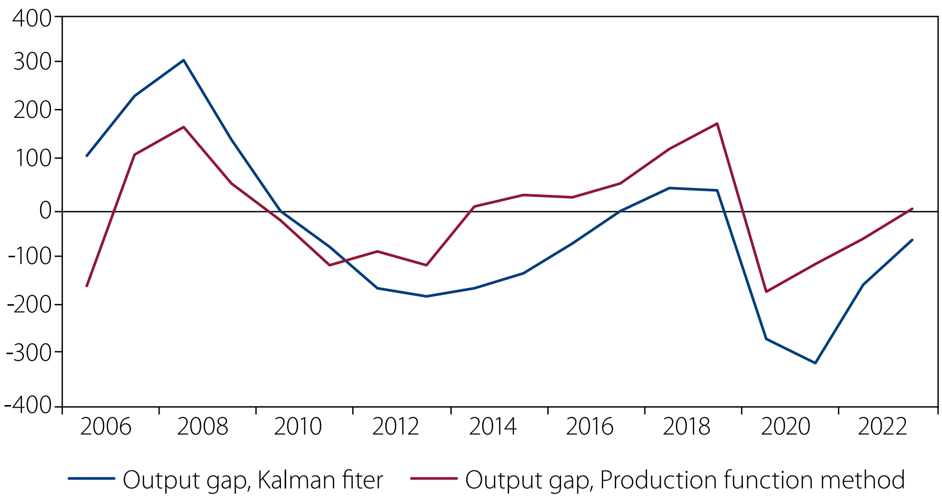
The results of the Kalman filter indicate that the gap is also significant in 2021, for the reason that the growth of the potential GDP is higher than the growth of the real GDP. However, when interpreting the results of the potential growth assessment, it is necessary to keep in mind a certain degree of uncertainty in the sense that the pandemic is still ongoing and that we use short-term projections of the European Commission, which affects the accuracy of the assessment. A similar interpretation is given in the ECB Economic Bulletin (2020). Namely, the authors of this bulletin expect revisions of potential output for the euro area countries once the size and consequences of the crisis have become clearer. The results indicate that in 2022 and 2023, the negative gap between potential and real GDP will be reduced, which means that in the coming period the economy will recover more strongly and function closer to its maximum potential. Thus, just as in the case of the global financial crisis, the crisis caused by the COVID-19 pandemic has a strong negative impact on both potential and real GDP, as well as on the negative GDP gap. The duration of the negative GDP gap can be attributed to the high level of unemployment. Specifically, the number of unemployed in December 2021 is higher by 20.8% compared to December 2020, while compared to December 2019 it is higher by 52.6%.

GDP gap: a comparison of the two methods

A comparison of the output gap estimated by using the Kalman filter and the production method indicates clear similarities, especially when observing its trend. Both series point to a positive GDP gap in the period preceding the global financial crisis and just before the crisis caused by the COVID-19 pandemic. However, the positive and negative output gaps are more pronounced when estimated by the Kalman filter method, with the exception of the two-year period before the pandemic. Certain smaller quantitative differences between the methods used are still expected, bearing in mind that these are univariate and multivariate methods.

It is important to stress that both models indicate that the effects of the crisis, both the global financial crisis and the COVID-19 crisis, are pronounced and long-lasting, which is the conclusion given in other working papers (Fontanari et al., 2019). Same as Mc Morrow et al. (2021) we can also conclude that the negative GDP gap is shorter after the crisis caused by the COVID-19 pandemic than after the global financial crisis. Mc Marrow et al. (2021) conclude that the potential level of GDP will return to the pre-pandemic level faster and that long-term effects are possible only if a part of the labour force withdraws from the labour market, and with lower investments and capital obsolescence and/or bankruptcies in the coming period. Assuming that the coronavirus pandemic will be fully controlled in the analysed period, our paper also indicates that potential GDP and real GDP will be almost equal by 2023, in the absence of the aforementioned negative economic circumstances.

Graph 7 – Output gap: Production mathod and the Kalman filter method



Source: Authors' calculations

We can conclude that both methods give reliable and very logical results although each has its own drawbacks. The disadvantages of the Kalman filter are its sensitivity to assumptions about the initial parametres, and not so many links with the theoretical economic framework. On the other hand, the production method is based on economic theory, which allows the estimation of total factor productivity, which is important for understanding technological advancements. Nevertheless, this method uses the Cobb-Douglas production function, which is based on the assumptions of constant returns, technological progress that gradually changes over time, and applies the HP filter, which indicates the shortcomings of this methodological approach.

Given the different nature of these two methods, it is encouraging that they give very similar results which, points to high validity of their application in the economic policies of Montenegro. Moreover, both methods support the conclusions of recent studies on different durations of the negative GDP gap after the two economic crises in 2009 and 2020.

5. Concluding remarks and policy recommendations

The assessment of potential output and output gap has been an unavoidable subject of central bank macroeconomic analyses. Estimates of these two indicators are primarily used as a measure of the current state of the business cycle and as guidelines for determining fiscal, monetary and macroprudential policies, so it is important to have relevant estimates of potential growth and output gaps. However, as these are variables that cannot be measured directly, their estimates are subject to some degree of uncertainty.

This working paper presents the results of estimating the potential growth and GDP gap for Montenegro in the period from 2006 to 2023 that have been obtained by using two different methodological approaches: the production function method and the Kalman filter method.

The results indicate that there are certain minor quantitative differences in the estimates depending on the applied method. With the Kalman filter method, the GDP gaps (both positive and negative) are slightly larger than those obtained by the production method. It is not uncommon for the application of different methodologies to assess the potential growth and GDP gap to result in different results of the analysed indicators (Jovičić, 2019). However, both methods point to similar trends: a positive GDP gap before the global financial crisis and the crisis caused by the COVID-19 pandemic, and a negative GDP gap after these negative macroeconomic shocks. Similarly, both methods suggest that real GDP takes longer to catch up with the potential GDP after the global financial crisis than after the COVID-19 crisis although the second data should be taken with caution as it is based on GDP forecasts. Other recent publications came to similar conclusion. Therefore, we can conclude that regardless of the different nature of these methods, a consistent and very credible result has been obtained and that both methods are useful for economic policy making.

The results of the production method analysis show that the contribution of factor productivity is very small, and even negative in some periods. This low level of factor productivity indicates the existence of structural issues in our economy that prevent more efficient use of the existing resources. Earlier studies showed a very weak upward trend in total factor productivity in the manufacturing and service industries of Montenegro, which indicates that the fourth technological revolution has not gained momemntum (Vujanović, 2020). Therefore, it is often concluded that the output gap in Montenegro, same as in other transition countries, largely reflects structural changes and to a lesser extent the cyclical component (Kastrati et al., 2017). In this context, policies aimed at identifying and addressing these issues could have a very positive impact on long-term economic growth.

In order to increase potential growth, it is necessary to implement structural reforms that must aim at improving business conditions, product and labour markets, and strengthening the capacity for innovations. It is in the empirical literature that increasing attention is being paid to structural reforms in helping to achieve stronger and more sustainable potential growth. Structural reforms can affect the growth of production through different channels: productivity, capital accumulation and employment. The results of the IMF's 2015 paper suggest that sustainable and well-coordinated structural reforms usually coincide with periods of improved productivity and macroeconomic performance. They also confirm that a wide range of structural reforms, including banking and financial sector reforms, labour market issues, infrastructure and legal system reforms, and the business climate have impact on macroeconomic performance. For a transition economy such as Montenegro, which still faces the issue of obsolete technologies, policies that promote research and development, innovation and training of employees in necessary skills for using latest technologies are important. Productivity can be increased through technological development, especially in manufacturing industry, but also in the services sector which already enjoys the benefits of digitalisation.

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